Jābir b. Aflaḥ on lunar eclipses[1]

José Bellver

Key Words: Astronomy, Greek Astronomy, Medieval Astronomy, al-Andalus, Jābir b. Aflaḥ, Ptolemy, *Almagest*, *Iṣlāḥ al-Majisṭī*, Eclipse Theory, Lunar Eclipses, Criticisms of Ptolemy's *Almagest*

Abstract

In his most important work, the *Iṣlāḥ al-Majisṭī* or *Improvement of the Almagest*, the Andalusian mathematician and theoretical astronomer, Jābir b. Aflaḥ, presents a list of criticisms of Ptolemy's *Almagest*, mainly of a mathematical nature. One of these is devoted to the computation of the magnitude and phases of lunar eclipses. Ptolemy uses plane trigonometry and some approximations that Jābir b. Aflaḥ contests. Ptolemy obtains the magnitude and phases for two particular cases – when the Moon is at its apogee and when it is at its perigee – and computes a table of interpolation for any other lunar anomaly. Jābir b. Aflaḥ avoids the need for tables of interpolation providing a slightly different method for computing the magnitude and phases of a lunar eclipse. In addition, he claims to have found an error in Ptolemy's method of interpolation. However, thanks to aquotation of the *Almagest* appearing in the *Iṣlāḥ al-Majisṭī*, we conclude that Jābir b. Aflaḥ's criticism is due to the fact that there is a section

[1] The present paper was prepared as part of the research programme "The evolution of Science in al-Andalus' society from the Early Middle Ages to the Pre-Renaissance", sponsored by the Spanish Ministry of Science and Innovation (FFI2008-00234/FILO) and the European Regional Development Fund (ERDF).

missing in his manuscript of the *Almagest*, and not to an error committed by Ptolemy, nor to a deficient translation of the *Almagest*.

Prolegomena

Jābir b. Aflaḥ al-Ishbīlī,[2] known as Geber filius Afflay Hispalensis in medieval Western Europe, was a mathematician and theoretical astronomer who most probably flourished in Seville in the first quarter of the 12th century.[3] Jābir b. Aflaḥ is a leading figure in medieval astronomy thanks to his most important work, the *Iṣlāḥ al-Majisṭī*, which was translated into Latin and Hebrew.[4] The *Iṣlāḥ al-Majisṭī* or *Improvement of the Almagest* is a rewriting of the *Almagest* in which he simplifies its mathematical structure by avoiding Menelaus's theorem and ignoring its more practical elements. He also presents some criticisms of the *Almagest*, mainly of a mathematical nature.

Some of Jābir b. Aflaḥ's criticisms are aimed at Ptolemy's procedure for computing the magnitude and phases of eclipses. Mainly, the criticisms are centred on certain statements by Ptolemy dealing with the problem of parallax while considering solar eclipses. However, one of his criticisms refers to the fact that Ptolemy uses a table of interpolation to account for the effect of the lunar anomaly in both solar and lunar eclipses. In order to avoid the use of this table, Jābir b. Aflaḥ provides a new method for computing eclipses which is slightly different from Ptolemy's. This paper describes this method and Jābir b. Aflaḥ's criticism of this point.

1. Introduction

The aim of the method discussed is to compute the magnitude and phases of both solar and lunar eclipses. In the case of solar eclipses, in addition to the method provided, the lunar parallax must also be taken into account. So the method discussed, as it is, is suitable for lunar eclipses, while it is also a part of a broader method for dealing with solar eclipses.

Ptolemy devotes chapters VI.7 to VI.9 of the *Almagest* to the study of the magnitude and the phases of both solar and lunar eclipses and presents some tables for obtain these values.[5]

The magnitude of an eclipse (m) is the number of digits of the diameter of the eclipsed body obscured at the mid-eclipse – a digit being the twelfth part of a diameter. To compute the magnitude, the value of the immersion

[2] For a general introduction on Jābir b. Aflaḥ, see R.P. Lorch [1975], "The Astronomy of Jābir b. Aflāḥ", *Centaurus*, Vol. 19, pp. 85-107 — reprinted in R.P. Lorch [1995a], *Arabic Mathematical Sciences: Instruments, Text, Transmission*, Aldershot, VI — an abridgement of his doctoral thesis: *Jābir ibn Aflaḥ and his Influence in the West* (Manchester, 1971); and forthcoming J. Bellver [2009],"El lugar del *Iṣlāḥ al-Maŷisṭī* de Ŷābir b. Aflaḥ en la llamada «rebelión andalusí contra la astronomía ptolemaica»", *al-Qanṭara*, Vol. 30, fasc. 1 (2009). Other papers by Lorch on the work of Jābir b. Aflaḥ, are R.P. Lorch [1976], "The Astronomical Instruments of Jābir ibn Aflaḥ and the Torquetum", *Centaurus*, Vol. 20, pp. 11-34 [reprinted in R.P. Lorch [1995a], XVI]; R.P. Lorch [1995c], "Jābir ibn Aflaḥ and the Establishment of Trigonometry in the West" in Lorch (1995a), VIII; R.P. Lorch [1995b], "The Manuscripts of Jābir's Treatise" in Lorch (1995a), VII; R.P. Lorch [2001], *Thābit ibn Qurra, On the Sector-Figure and Related Texts. Edited with Translation and Commentary*, Frankfurt am Main, pp. 387-90. Other scholars have studied aspects of Jābir b. Aflaḥ's work, such as N.M. Swerdlow [1987], "Jābir ibn Aflaḥ's interesting method for finding the eccentricities and direction of the apsidal line of superior planets" in D.A. King and G. Saliba (eds.) [1987], *From Deferent to Equant. A Volume of Studies in the History of Science in the Ancient and Medieval Near East in Honour of E.S. Kennedy*, New York, pp. 501-12; H. H. Hugonnard-Roche [1987], "La théorie astronomique selon Jābir ibn Aflaḥ", in G. Swarup, A.K. Bag and K.S. Shukla (1987), *History of Oriental Astronomy. Proceedings of an International Astronomical Union Colloquium nº 91 (1985)*, Cambridge, pp. 207-8; J. Samsó [2001], "Ibn al-Haytham and Jābir b. Aflaḥ's Criticism of Ptolemy's Determination of the Parameters of Mercury", *Suhayl*, Vol. 2 (2001), pp. 199-225 — reprinted in J. Samsó [2007], *Astronomy and Astrology in al-Andalus and the Maghrib*, Aldershot - Burlington, VII —; J. Bellver, "Jābir b. Aflaḥ on the four-eclipse method for finding the lunar period in anomaly", *Suhayl*, Vol. 6 (2006), pp. 159-248; J. Bellver [2007], "Ŷābir b. Aflaḥ en torno a la inclinación de los eclipses en el horizonte", *Archives Internationales d'Histoire des Sciences*, Vol. 57, Fasc. 158 (2007), pp. 3-25 and the forthcoming J. Bellver [2008-9], "Jābir b. Aflaḥ on the lunar eccentricity and prosneusis at syzygies", *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften*.

[3] See Lorch [1975], pp. 85-6.

[4] In this research I have mainly considered the only three extant Arabic manuscripts in Arabic script: Mss. Escorial 910 [henceforth referred to as Es[1]], Escorial 930 [henceforth referred to as Es[2]] and Berlin 5653 [henceforth referred to as B.]. In some sections of the *Iṣlāḥ al-Majisṭī*, Mss. Escorial 910 and Berlin 5653 differ, while Ms. Escorial 930 follows one or the other. On the *Iṣlāḥ al-Majisṭī*'s manuscripts, see Lorch, R.P. [1995b].

[5] See G.J. Toomer [1984], *Ptolemy's Almagest*, London [henceforth referred to as PtA], pp. 294-310; O. Pedersen [1974], *A Survey of the Almagest*, Odense, pp. 231-5; O. Neugebauer [1975], *A History of Ancient Mathematical Astronomy*, 3. vols., Berlin – Heidelberg – New York [henceforth referred to as HAMA], pp. 134-9 and 1240.

(μ) is needed. This is the angular distance between the rim of the eclipsing body and the point of the rim of the eclipsed body nearest the centre of the eclipsing body at the mid-eclipse.[6] See Figure 1 for an example of the value of immersion.

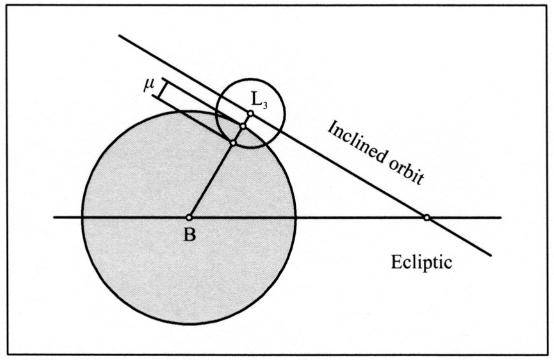

Figure 1. Example of immersion of the Moon

To compute the magnitude, when the immersion of the eclipsed body is complete, the value of immersion – μ – is equal to the diameter – d – of the eclipsed body, and thus the magnitude of the eclipse – m – must be 12 digits. Therefore, the value of the magnitude is

$$m = 12\,\mu\ /\,d = 6\,\mu\ /\,r \qquad (1.)$$

where d is the diameter of the eclipsed body and r its radius.

As for the phases of the lunar eclipse, Ptolemy considers two values: the minutes of immersion and the half of totality. The minutes of immersion correspond to the arc from the beginning of the eclipse to the beginning of totality, while the half of totality corresponds to the arc between the beginning of totality and the middle of the eclipse. So let us consider Figure 2, where point A is the centre of the shadow cone, and points Z, E, D, G and B refer respectively to the beginning of the eclipse, the beginning of totality, the middle of the eclipse, the end of totality and the end of the eclipse; and let us consider that the phase of immersion is equal to the phase of emersion.

[6] See HAMA p. 135 and pp. 1241-2.

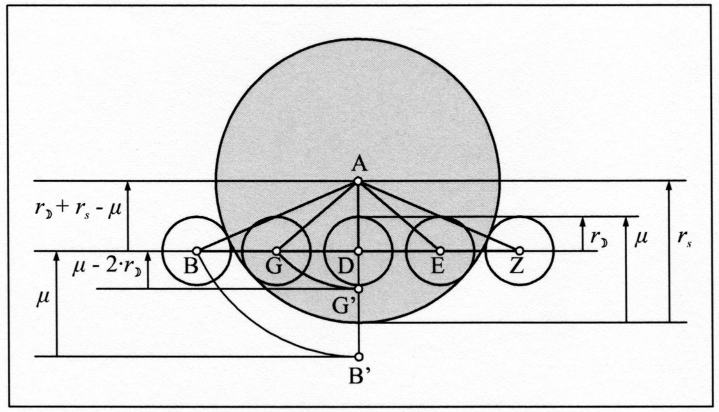

Figure 2. Phases of a lunar eclipse

Knowing from (1.) that $\mu = m\,r_{☽}\,/\,6$, the half of totality, GD, for a given magnitude, m, is

$$GD = [\,(\,r_s - r_{☽}\,)^2 - (r_s + (\,1 - m/6\,)\,r_{☽}\,)^2\,]^{1/2} \qquad (2.)$$

and the minutes of immersion, BG, for a given magnitude is

$$BG = BD - GD \qquad (3.)$$

provided that

$$BD = [\,(\,r_s + r_{☽}\,)^2 - (r_s + (\,1 - m/6\,)\,r_{☽}\,)^2\,]^{1/2} \qquad (4.)$$

where r_s is the radius of a section of the shadow cone of the Earth and $r_{☽}$ is the radius of the Moon.

Ptolemy provides five tables in *Almagest* VI.8 to compute the magnitude and phases of both solar and lunar eclipses. The design of these tables is explained in *Almagest* VI.7. Two tables are devoted to solar eclipses, and the other two to lunar eclipses. Ptolemy considers two tables for each kind of eclipse since he takes two cases into account: when the Moon is in its apogee and when it is in its perigee. These tables are tabulated using as argument the true argument of latitude, and provide the values of the magnitude and the duration of the phases. Once these values are obtained for both cases – that is, when the Moon is located in its apogee and when it is in its perigee, Ptolemy uses a fifth table to obtain a coefficient of interpolation – $q(\alpha)$ – between the two cases according to the value of the lunar anomaly (α) which ranges from 0° to 180°.

So let us consider the magnitude of an eclipse. Suppose that m_a and m_p are the magnitudes of an eclipse when the Moon is in its apogee and in its perigee. The magnitude – m – for a given anomaly – α –is

$$m = m_a + (m_p - m_a)\, q(\alpha) \qquad (5.)$$

where $q(\alpha)$ is the coefficient of interpolation obtained with Table V.

Similarly, let us consider the duration of a phase of an eclipse – η . Suppose that η_a and η_p are the values of the duration of an eclipse when the Moon is at its apogee and at its perigee. The duration of the phase – η – for a given anomaly – α – is

$$\eta = \eta_a + (\eta_p - \eta_a)\, q(\alpha) \qquad (6.)$$

where q(α) is the coefficient of interpolation obtained with Table V.

In addition, Ptolemy considers what happens when some arguments of latitude appear in table IV, but not in table III; that is, there are some arguments of latitude in which, when the Moon is at its perigee, an eclipse can take place, but not when it is at its apogee. The intervals of arguments of latitude in this situation are the following:

{ [77;48°, 79;12°], [100;48°, 102;12°], [257;48°, 259;12°], [280;48°, 282;12°] }

In this case, Ptolemy points out that in (5.) and (6.) m_a and η_a must be zero and therefore the values of the magnitude and phases under these conditions are:[7]

$$m = m_p\, q(\alpha) \qquad (7.)$$
$$\eta = \eta_p\, q(\alpha) \qquad (8.)$$

[7] Ptolemy points out: "*If, however, it happens that the argument of latitude falls within the range of the second table only, we take* [*as final result*] *the appropriate fraction (determined by the number of sixtieths found* [*from the correction table*]*) of the digits and minutes* [*of travel*] *corresponding to the argument of latitude in the second table alone.*" In this quotation, the second table refers to the one computed when the Moon is at its perigee. See PtA p. 306.

when there is no entry in table III for the maximum lunar distance.

2. Jābir b. Aflaḥ's method

Jābir b. Aflaḥ follows Ptolemy's method for computing the value of the magnitude and the duration of the phases closely, except for a few small differences that he introduces. The main one is that he provides a way to account for the effect of the anomaly on the lunar radius before obtaining the value of the magnitude and the duration of the phases, whereas Ptolemy, as we have seen, accounts for it after obtaining these values.

Previous elements

Jābir b. Aflaḥ points out some previous elements needed to solve the magnitude and phases of an eclipse. In fact, it is in the consideration of these previous elements that Jābir b. Aflaḥ improves Ptolemy's method.

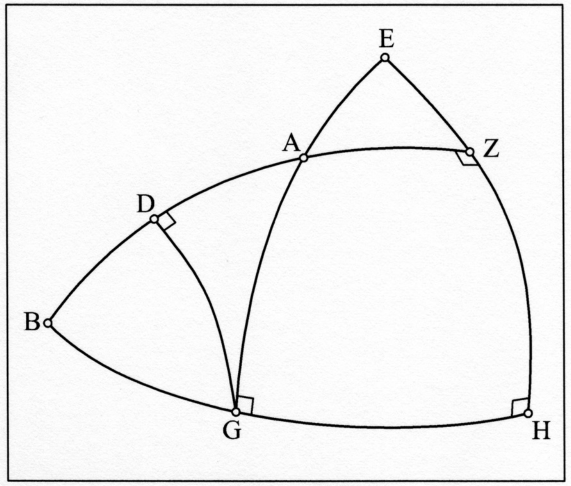

Figure 3. Ms. Es[1] 62v.

Firstly, Jābir b. Aflaḥ endeavours to obtain the arc of great circle between the centre of the Moon and the centre of the shadow cone – for lunar eclipses – or the centre of the Sun – for solar eclipses. Jābir b. Aflaḥ relies upon Figure 3 where point B is the lunar node, circle BH is the ecliptic and circle BZ is the lunar inclined orbit, point G is the centre of the Sun (or of the shadow cone) at the true syzygy, point A is the centre of the

Moon at the true syzygy, point D is the centre of the Moon at the mid-eclipse and point E is the pole of the ecliptic. In addition, arc GD is perpendicular to the lunar incline orbit – circle AB – arc AG is perpendicular to the ecliptic – circle BG – and arc AZ is perpendicular to circle EZ while circle EZ passes through the pole of the ecliptic and the pole of the lunar inclined orbit. Hence, point Z is the point of the inclined orbit with maximum latitude which can be found at 90º from the node and Jābir b. Aflaḥ calls it "maximum inclination of the inclined orbit" (*nihāyat mayl al-falak al-mā'il*).

Jābir b. Aflaḥ obtains arc GD – which is the arc of the great circle between the centre of the Moon and the centre of the Sun (or the shadow cone) at the mid-eclipse – in order to obtain the magnitude of the eclipse – which is a sector of arc GD. Therefore, he needs to know arc AG – which is the latitude of the Moon at the true syzygy. Ptolemy assumes that the value of the two is approximately equal since the difference between AG and GD amounts to two minutes. Jābir b. Aflaḥ considers that this difference amounts to four minutes, and criticises Ptolemy for his assumption. This minor criticism is dealt with in the next section of this paper.

Jābir b. Aflaḥ describes the procedure for obtaining arc AG as follows:

> As for the value of arc AG – which is the lunar latitude at the true syzygy –, it is known since arc AB – which is the lunar nodal distance in the inclined orbit – is known. And the ratio of its sine to sine of arc AG is equal to the ratio of the radius to the sine of the maximum inclination of the inclined orbit (*nihāyat mayl al-falak al-mā'il*).[8]

Jābir b. Aflaḥ applies the rule of four quantities or the sine law, since both, in this case, are equivalent. By radius, he means the radius of a generic circle; i.e. 60^p. Since Sin BZ = 60^p, the sine of the maximum inclination of the inclined orbit – i.e. the maximum latitude of the inclined orbit – corresponds to the sine of arc ZH. Therefore, the procedure is as follows:

[8] Cf. *infra* p. 85.

∵ AB known

∵ Sin AB / Sin AG = Sin BZ / Sin ZH with Sin BZ = 60^p (if we consider that he is applying the rule of four quantities)

or

Sin AB / Sin AG = Sin ∠G / Sin ∠B — with Sin ∠G = 60^p (if we consider that he is applying the sine law)

∴ AG known — with AG < 90º since Sin ZH = Sin *i*

where *i* is the angle between the lunar inclined orbit and the ecliptic. To obtain GD, Jābir b. Aflaḥ applies the rule of the four quantities as follows:

∵ ∠Z = ∠D = 90º

∵ ∠EAZ = ∠GAD

∴ Sin AG / Sin GD = Sin AE / Sin ZE

∵ Sin AG, Sin AE and Sin ZE known — since AE = 90º – GA and ZE = 90º – HZ

∴ Sin GD known — with GD < 90º

This is the procedure for obtaining the distance between the centre of the Moon and the centre of the Sun (or the shadow cone) at mid-eclipse.

In order to ascertain the value of the difference between AG and GD, let us consider, according to Ptolemy, that the inclination of the lunar inclined orbit relative to the ecliptic (i = ∠ABG) is 5º and that the maximum value of BA – the maximum nodal distance along the inclined orbit in which an eclipse can take place – is 12º. We know that

tan BG = tan BA cos *i*
tan BD = tan BG cos *i*

and thus

BG = 11;57,20° and BD = 11;54,41°

so the difference we are looking for is

DA = BA – BD = 0;5,19°

This difference is slightly greater than the one indicated by Jābir b. Aflaḥ.[9] Later, Jābir b. Aflaḥ continues by obtaining the diameter of the Moon and the shadow cone of the Earth for any lunar anomaly.[10] To do so, he provides two figures which are described in Figure 4. The right-hand figure shows a plane epicycle where circle ABG around centre D is the lunar epicycle, point E is the centre of the Earth, point A is the lunar apogee and point B its perigee, point G is the centre of the Moon, line GZ is perpendicular to line ADBE and distance EG is equal to distance EH. Jābir b. Aflaḥ wants to obtain the diameter of the Moon and of the shadow cone for distance EG.

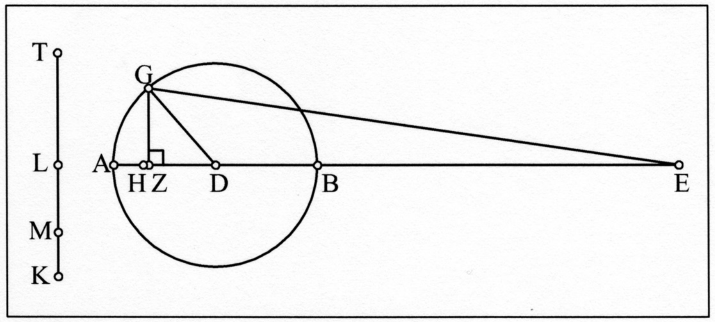

Figure 4. Ms. Es[1] 63r.

The left-hand figure shows a scale of the diameters where TK corresponds to the apparent diameter of the Moon at the perigee and TL corresponds to

[9] See PtA p. 298 n. 55 and HAMA p. 83 n. 5.

[10] See PtA pp. 283-5 for Ptolemy's method to obtain the maximum radius of the Moon and of the shadow cone of the Earth, and PtA pp. 252-4 for his method to obtain the minimum radius of the Moon.

its apparent diameter at the apogee. Line TM corresponds to the apparent diameter of the Moon at point G. Jābir b. Aflaḥ wants to know the value of line TM.

Firstly, he endeavours to obtain the distance between the centres of the Moon and the Earth, knowing that the distance between the centre of the epicycle and the centre of the Earth is 60^{P} and that the diameter of the epicycle is 10^{P}. Jābir b. Aflaḥ's procedure is as follows:

∵ AB = 10^{P} and ED = 60^{P}

∵ ∠ADG known — since ∠ADG is the given anomaly

∴ Sin ∠ADG known

∴ AZ = Vers ADG known

∴ EZ = EA –ZA known

∴ EG known

EG is known since, using a modern function,

∠ZEG = arctan (GZ / ZE)
EG = EZ / cos (∠ZEG)

And finally, Jābir b. Aflaḥ ends this first part of the resolution of the lunar diameter for any lunar anomaly indicating that:

∴ EH = EG known

Once he has found the lunar distance to the centre of the Earth, Jābir b. Aflaḥ continues with the resolution of the lunar diameter for any lunar anomaly. He bases his resolution upon the left-hand figure of Figure 4. which shows the proportion of the lunar diameters as related to the lunar anomaly. Jābir b. Aflaḥ's procedure is as follows:

∵ Lunar diameters at the apogee – TL – and at the perigee – TK – known.

$\because$ LK = TK – TL known

$\because$ AB = AE – BE known

$\because$ AH = AE – HE known

$\because$ AH / AB $\cong$ ML / LK

$\therefore$ LM known

$\therefore$ TM = TL + LM known

So in order to obtain the lunar diameter relative to the anomaly, Jābir b. Aflaḥ considers that the lunar diameter decreases linearly with distance. This allows him to establish a proportionality between, on the one hand, the maximum and minimum diameters pertaining to the perigee and the apogee and, on the other, the diameter relative to any other lunar anomaly. Jābir b. Aflaḥ seems well aware of the fact that the epicyclic hypothesis does not explain the actual diameter of the Moon at its perigee. Hence he only considers the observed values of the lunar diameter when the Moon is at its apogee and its perigee, not the values computed with the hypothesis. Therefore he bases the aforementioned proportionality upon differences in the lunar diameter when the Moon is at its apogee and its perigee, thus avoiding the use of distances related to the centre of the Earth. This is the main difference between Jābir b. Aflaḥ's method and Ptolemy's.

Jābir b. Aflaḥ uses the proportion of the difference between the lunar diameter at the apogee and the perigee – line LK – and the difference between the lunar distance at the perigee and the apogee – line AB. In this way, he obtains the difference in the lunar diameter for a given anomaly relative to its diameter at the apogee – ML – from the difference between the lunar distance for the given anomaly and the lunar distance at the apogee – AH. By this trick, he avoids using the difference between the observed lunar diameter at the perigee and the computed one.

Finally, Jābir b. Aflaḥ points out, first, that the method for computing the diameter of the shadow cone is the same as the one shown for the lunar diameter and, second, that the solar diameter, which is slightly affected by the parallax, is equal to the lunar diameter when the Moon is at its apogee.

On the magnitudes and phases of lunar eclipses

After considering the procedures for finding the elongation between the lunar and solar centres at the mid-eclipse and the diameters of the Moon and of the shadow cone for any anomaly, Jābir b. Aflaḥ investigates the magnitude of lunar eclipses and the duration of their phases according to Figure 5, where arc AB is the ecliptic, arc GB is the lunar inclined orbit and point A is the centre of the Sun at the solar eclipse and the centre of the shadow cone at the lunar eclipse.

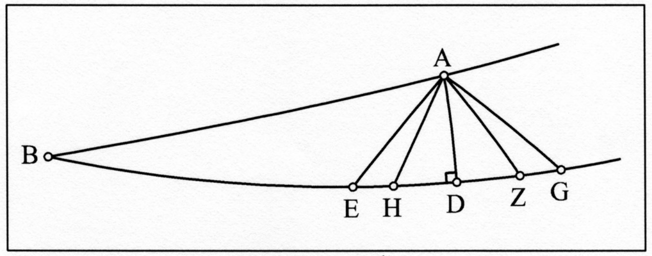

Figure 5. Ms. Es^{1} 63v.

Jābir b. Aflaḥ considers three significant points which determine the phases of solar eclipses – the beginning, middle and end of the eclipse – and five significant points which determine the phases of lunar eclipses – the previous three points plus the beginning of totality and the end of totality. The common significant points for solar and lunar eclipses are point G, which is the lunar centre at the beginning of the eclipse, point D, which is the lunar centre at the mid-eclipse and point E, which is the lunar centre at the end of the eclipse. In addition, point Z corresponds to the beginning of totality, and point H to the end of totality.

Jābir b. Aflaḥ establishes the next set of conditions. Firstly, he points out that arc AG is equal to the sum of the radius of the Sun and Moon at the solar eclipse, or to the sum of the radius of the Moon and the shadow cone at the lunar eclipse. Moreover, he also establishes that arc AD is perpendicular to arc BG. And finally, he points out that arc AZ is equal to the radius of the shadow cone.

Additionally, he considers that the different phases of an eclipse are symmetrical as related to the mid-eclipse – to put it with modern terminology. So arc DE is equal to arc GD, arc GZ is approximately equal to arc HE and arc ZD is approximately equal to arc DH.

Once he has described this figure, he tries to obtain the magnitude of the eclipse. The procedure is as follows.

$\because$ The lunar latitude at the mid-eclipse —β(☽)— is known

$\therefore$ AD known.

$\therefore$ $d_{☽} + d_s - \text{AD}$ known

$\therefore$ magnitude of the eclipse known

where $d_{☽}$ is the diameter of the Moon and d_s is the diameter of the shadow cone. In order to obtain the magnitude of the eclipse, he considers the distances obtained in the previous section. The lunar magnitude at the true syzygy corresponds to side AG of Figure 3, while arc AD corresponds to the arc between the centre of the Moon and the centre of the shadow cone i.e. arc GD in the same figure. Next he subtracts arc AD from the sum of the diameters obtained in the previous section and points out that the magnitude is therefore known.

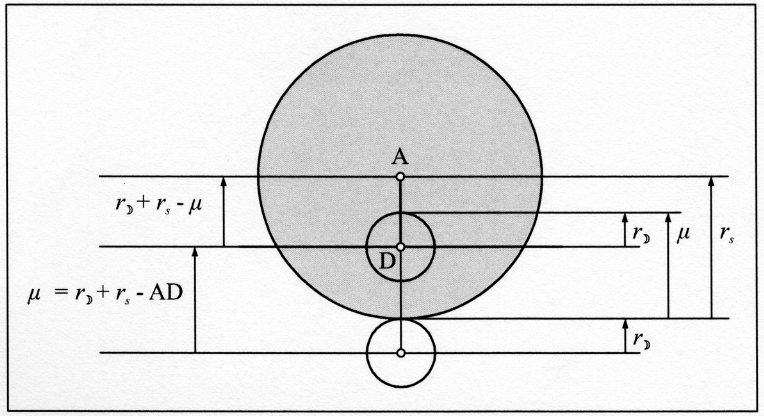

Figure 6. Value of the immersion from the radius of the Moon and the shadow cone

But from Figure 6, the value of immersion is obtained by subtracting arc AD from the sum of the radius of the Moon and of the shadow cone, not

from the sum of the diameters as Jābir b. Aflaḥ suggests.[11] Maybe this lack of thoroughness is due to the fact that he is indicating the procedure of resolution – though this would be out of character in his case – or it may be a textual error. The Arabic text is as follows:

> فنسقطها من مجموع القطرين يبقى مقدار المنكسف من قطر المكسوف معلوما[12]
>
> "And we subtract it from the sum of both diameters and therefore the magnitude is known"

In addition, I cannot find textual variants between the different manuscripts. Nor does the text of the Latin translation by Gerard of Cremona published by Petrus Apianus differ from the Arabic manuscripts:

> Proijciam ergo ipsum de aggregatione duarum diametrorum et remanebit quantitas eclipsantis de quantitate eclipsati nota.[13]

In any case, the immersion must be:

$$\mu = r_{☽} + r_s - \text{AD} \qquad (9.)$$

Once the immersion is obtained, the magnitude is as

$$m = 6\,\mu\,/\,r_{☽} = 6\,(\,r_{☽} + r_s - \text{AD}\,)\,/\,r_{☽} \qquad (10.)$$

After obtaining the eclipse magnitude, Jābir b. Aflaḥ considers, without further indications, the value of the arcs related to the phases of the

[11] In fact, when he considers this point for solar eclipses, he points out that, to obtain the magnitude of the eclipse, the distance between the solar and lunar centres must be subtracted from the sum of the solar and lunar radius.

[12] Cf. *infra* p. 79.

[13] Petrus Apianus, *Instrumentum primi mobilis. Accedunt iis Gebri filii Affla Hispalensis Astronomi vetustissimi pariter et peritissimi, libri IX de astronomia, ante aliquot secula Arabice scripti, et per Giriardum Cremonensem latinitate donati, nunc vero omnium primum in lucem editi*, Nuremberg, 1534, p. 78.

eclipse. Firstly, he obtains arc GD after stating that it is approximately equal to arc DE (see Figure 6). The procedure is as follows:

∵ AG known	since $AG = r_☽ + r_s$
∵ AD known	as demonstrated in the introductory section
∵ $\angle D = 90°$	
∴ GD is known.	

GD is actually known since

$$GD = \arccos [\, \cos AG \cdot \cos AD \,]$$

Lastly, Jābir b. Aflaḥ seeks to solve the arcs related to the intermediate phases: arc GZ, which is the arc between the initial eclipse and the beginning of totality, and arc ZD, which is the arc between the beginning of totality and the mid-eclipse.

∵ AZ known	since $AZ = r_s$
∵ AD known	as demonstrated in the introductory section
∴ ZD known	

since, as above, we can apply

$$ZD = \arccos [\, \cos AZ \cdot \cos AD \,]$$

So Jābir b. Aflaḥ continues to assume that ZD is know and that

∵ GD known	
∴ GZ known	since GZ = GD – ZD.

In short, Jābir b. Aflaḥ's procedure for obtaining the arcs relative to the phases is similar to Ptolemy's when he considers the situations when the Moon is at the apogee and the perigee, although Jābir b. Aflaḥ applies spherical trigonometry and Ptolemy the theorem of Pythagoras.

Jābir b. Aflaḥ's method for obtaining the magnitude of the eclipse is slightly different from Ptolemy's. First, he avoids considering that the arc between the centre of the shadow cone and the centre of the Moon at the mid-eclipse is equal to the latitude of the Moon at the apparent syzygy. He also uses spherical trigonometry throughout his method. But the main difference is that Jābir b. Aflaḥ considers the lunar anomaly before computing the magnitude and phases of the eclipse, while Ptolemy uses an interpolation table for any lunar anomalies, thus considering the lunar anomaly after computing the magnitude and phases of lunar eclipses while the Moon is at the perigee and the apogee.

3. Jābir b. Aflaḥ's criticisms of Ptolemy

Jābir b. Aflaḥ criticises two points of Ptolemy's approach to lunar eclipses. The first criticism centres upon Ptolemy's premises that he considers to be approximate. Jābir b. Aflaḥ states:

> As to the procedure mentioned by Ptolemy, it is an approximate procedure from two points of view. Firstly, he used straight lines instead of arcs. And secondly, he considered that the arc between the centres of the two bodies – the eclipsing and eclipsed bodies – at the middle of the eclipse was equal to the true latitude of the Moon at the true syzygy.[14]

That is, Jābir b. Aflaḥ points out that (i.) Ptolemy uses plane trigonometry and that (ii.) he considers that the difference between the position of the Moon at the mid-eclipse and its position at the apparent syzygy is negligible. However, it must be remembered that Ptolemy was well aware that his procedure was approximate.

Jābir b. Aflaḥ corrects the value of the error due to the approximation, stating:

[14] Cf. *infra* p. 89.

> He [i.e. Ptolemy] considered that [the difference between the centres under these conditions] was of two minutes.[15] However, [this difference] is greater than this value.[16]

Jābir b. Aflaḥ considers this difference to be 4 minutes.[17] The cause of this difference is not that Ptolemy bases his computation on apparent syzygies while Jābir b. Aflaḥ uses true ones since the parallax does not affect lunar eclipses, but the fact that Jābir b. Aflaḥ is using spherical trigonometry. This difference has been computed in the previous section of this paper.

The second point Jābir b. Aflaḥ criticises involves some difficulties derived from the use tables computed for the lunar apogee and perigee and the need to interpolate the results for a given anomaly.

Jābir b. Aflaḥ describes first the method of Ptolemy for computing the magnitudes and phases of the lunar eclipse. He then describes how to use the table of interpolation.[18] He quotes the *Almagest*:

> Enter the value of the [argument in] latitude in both tables and take what you find in front of both [tables] and write it down separately. Enter the degrees of anomaly in the row of the [corresponding] value in the table of minutes and take the value of minutes that you find in front of it. Take a value from the difference of the values obtained from both tables such that its proportion to [this difference] is equal to the [proportion of the minutes obtained relative to 60']. This [proportion] is added to the value obtained from the first table. The result in digits corresponds to the eclipsed section of the lunar diameter.[19]

This quotation describes equations (5.) and (6.) where Ptolemy summarizes his method of interpolation.

[15] Cf. PtA p. 297-8.

[16] Cf. *infra* p. 89.

[17] Cf. *infra* p. 86.

[18] See the Table of Correction in PtA p. 308.

[19] Cf. *infra* p. 90, and also PtA pp. 305-6.

The quotation of the *Almagest* in the *Iṣlāḥ al-Majisṭī* continues:

> In case the value of the [argument of] latitude does not exist in the first table, but only in the second, take the digits that you find opposite it. This [value] corresponds to the magnitude of the lunar eclipse (*miqdār al-munkasif min quṭr al-qamar*).[20]

In the last quotation, the following situation is considered: when, for the same lunar argument of latitude, the lunar eclipse occurs when the Moon is at its perigee, but not when it is at its apogee. From this quotation of the *Almagest*, as it appears in the *Iṣlāḥ al-Majisṭī*, whenever a lunar argument of latitude is not found in the table for the maximum lunar distance, but only in the table for the minimum lunar distance, the table of interpolation must not be considered. In this situation, from this quotation as it appears in the *Iṣlāḥ al-Majisṭī*, equations (7.) and (8.), should be

$$m = m_{\mathrm{P}} \tag{11.}$$
$$\eta = \eta_{\mathrm{P}} \tag{12.}$$

Hence, whenever a lunar eclipse occurs when the Moon is at its apogee, but not when it is at its perigee, the magnitude and phases of the eclipse are independent of the anomaly and should be considered as if the Moon were at the perigee. Jābir b. Aflaḥ's criticism follows the above quotation. He states:

> If the lunar [argument in] latitude were 79º, we will not find [an entry] in the first table. However, in the second table in front of the [argument of latitude of 79º], we will find a value greater than two digits.
>
> [But] we said that the eclipsed section of the Moon is greater than two digits. And this is only so when the Moon is at the perigee of its epicycle. If the Moon were at the apogee or next to it, in such a way that the addition of the two diameters was equal to the lunar latitude or less than it, no part of the Moon will be eclipsed. But we have stated that it is eclipsed more than two digits. And his pretension that the eclipsed section of the Moon

[20] Cf. *infra* p. 90.

> is greater than a sixth is extremely clumsy for it is not eclipsed at all.[21]

The argument of latitude of the first row of the table for the maximum lunar distance is 79;12º. Therefore there is no entry for an argument of latitude of 79º. Instead, in the second table, the argument of latitude 78;56º corresponds to a magnitude $m_p = 2^d$. So, according to the quotation of the *Almagest* found in the *Iṣlāḥ al-Majisṭī*, equation (11.) can be applied, since there is no corresponding argument of latitude in the table for the maximum lunar distance. In this case, since the magnitude does not depend on the anomaly, when the Moon is located in anomalies next to the apogee, the eclipsed magnitude is $m = 2^d$; however, it must be 0^d as Jābir b. Aflaḥ states.

After noting this fact, Jābir b. Aflaḥ goes on to make a fierce criticism[22] based upon two facts. First, he considers that Ptolemy is inconsistent since, on the one hand, he points out imperceptible minutiae – as when he considers that the immersion and emersion times of an eclipse are different[23] – and, on the other, he does not account for a fact that provides a difference of two digits in the magnitude of an eclipse. The second point criticised by Jābir b. Aflaḥ is based the fact that Ptolemy made the computation of these functions excessively complicated.

In any case, the origin of the error pointed out by Jābir b. Aflaḥ must be examined. Firstly, the quotation of the *Almagest* in the *Iṣlāḥ al-Majisṭī* differs from Toomer's version. He translates:

> If, however, it happens that the argument of latitude falls within the range of the second table only, we take [as final result] the appropriate fraction (determined by the number of sixtieths found [from the correction table]) of the digits and minutes [of travel] corresponding [to the argument of latitude] in the second table alone. The number of digits which we find as a result of

[21] Cf. *infra* p. 91.

[22] *Ibidem*.

[23] See the forthcoming Bellver [2009] for a summary of his criticism of this point.

> the above correction will give us the magnitude of the obscuration, in twelfths of the lunar diameter, at mid-eclipse.[24]

That is, Ptolemy considers equations (7.) and (8.) which I copy here:

$$m = m_p\, q(\alpha)$$
$$\eta = \eta_p\, q(\alpha)$$

in which a correction as a function of the anomaly is taken into account, and not equations (11.) and (12.) which have been derived from the quotation of the *Almagest* as it appears in the *Iṣlāḥ al-Majisṭī*. So Jābir b. Aflaḥ's criticism seems to be unjustified since it derives from a possible textual error in the manuscript of the *Almagest* he is using. Let us consider this point.

The text of Isḥāq b. Ḥunayn's translation is as follows:

> وإن اتّفق أن يقع عدد العرض في الجدول الثاني وحده وضعنا ما يوجد فيه وحده بإزاء ذلك العدد من الأصابع والدقائق فما خرج لنا من الأصابع من هذا التقويم قلنا إنّ {على}[25] عدد تلك الأصابع يكون عدد أجزاء من اثنى عشر جزءًا يحيط بها الظلمة من قطر القمر في الزمان الوسط من الكسوف[26]

While in al-Ḥajjāj's translation we read:

> فإن اتّفق أن يقع عدد العرض في الفصل الثاني فقط أثبتنا الدقائق الموجودة التي هي للأصابع والأجزاء التي تقابل موضعه وحده فكلّ

[24] Cf. PtA p. 306.

[25] In the margin.

[26] Version of Isḥāq b. Ḥunayn according to Ms. BN Paris Ar. 2482 f. 123v.

ما وجدناه خرج لنا من الأصابع من هذا التقويم قلنا إنّ عدد تلك
الأصابع يكون عدد أجزاء من اثنى عشر جزءا تحيط بها الظلمة من
قطر القمر في الزمان الأوسط من الكسوف[27]

Finally, the text of the quotation which appears in the *Iṣlāḥ al-Majisṭī* is the following:

وإن كان عدد العرض لا يوجد في الجدول الأوّل ولاكن يوجد في
الثاني وحده أخذ ما بإزائه منه من الأصابع فكان ذلك مقدار المنكسف
من قطر القمر. هذا هو العمل الذي ذكره[28]

From the collation of the previous texts, it can be concluded that Jābir b. Aflaḥ is not quoting literally any of the translations of the *Almagest* available to him.[29] Instead, he is adapting the text or quoting a previously adapted text. He is clearly adapting the terminology, since the long periphrasis

عدد أجزاء من اثنى عشر جزءا تحيط بها الظلمة من قطر القمر في
الزمان الأوسط من الكسوف

[27] Version of al-Ḥajjāj according to Ms. British Museum Add. 7474 f. 176v.

[28] Cf. *infra* p. 82.

[29] For the quotation of Jābir b. Aflaḥ's in which he states that he had consulted Isḥāq b. Ḥunayn and al-Ḥajjāj's translations of the *Almagest*, see Lorch [1975], pp. 96-7, especially n. 61. See my forthcoming Bellver [2008, forthcoming], for a discussion of Jābir b. Aflaḥ's criticism in which he makes this comment and also Bellver [2009, forthcoming] for a discussion of his degree of acquaintance with both translations.

which is almost identical in both translations is rendered in the *Iṣlāḥ al-Majisṭī* as *miqdār al-munkasif min quṭr al-qamar*, which can be translated as 'magnitude of the lunar eclipse'.

Secondly, from an analysis of the texts, it is possible that the version of the *Almagest* used by Jābir b. Aflaḥ is, as usual, Isḥāq b. Ḥunayn's translation, since he uses certain expressions and terms that only appear in this version; i.e., in the *Iṣlāḥ al-Majisṭī* we find the term *jadwal*, which is used in Isḥāq b. Ḥunayn's version, but not in al-Ḥajjāj, who uses *faṣl* instead. Another case is the expression *bi-'izā'* (opposite) which in al-Ḥajjāj appears as *allatī tuqābilu mawḍiʿa-hu.*

Here is a translation of Isḥāq b. Ḥunayn's version:

> And if it happens that the argument of latitude only appears in the second table, the digits and minutes found will be placed opposite [the argument of latitude]. [As for] the digits obtained after this correction (*fa-mā kharaja lanā min al-aṣābiʿ min hādhā al-taqwīm*), we said: the number of these digits corresponds to the number of the parts relative to twelve of the lunar diameter obscured at the mid-eclipse.

A comparison of this version of the *Almagest* with that of Jābir b. Aflaḥ shows clearly that the author of the *Iṣlāḥ al-Majisṭī* did not take into account the sentence "[As to] the digits obtained after this correction" (*fa-mā kharaja la-nā min al-aṣābiʿ min hādhā l-taqwīm*). From this sentence, it can be concluded that the digits corresponding to the magnitude of the lunar eclipse, when an argument of latitude is found in the table for the lunar perigee and not in the table for the lunar apogee, have been corrected. In addition, from the context, it must be assumed that the correction is performed using the table of interpolation. In this particular case, the sense of al-Ḥajjāj's version does not differ from that of Isḥāq b. Ḥunayn. In conclusion, despite the fact that the text is not completely clear, equations (7.) and (8.) fit better the description found in the Arabic versions.

Given that Jābir b. Aflaḥ usually reads the *Almagest* with great care, it is unlikely that he forgot to mention the word *taqwīm* (equation, correction). In addition, it does not make sense to base his fierce criticism of Ptolemy on a careless reading. Therefore, the most likely hypothesis is that the words *min hādhā l-taqwīm* did not appear in the manuscript of the *Almagest* Jābir b. Aflaḥ used when he wrote the *Iṣlāḥ al-Majisṭī*.

However, the fact that Jābir b. Aflaḥ's criticism is not justified does not imply that Ptolemy's procedure is completely correct, assuming the presuppositions of the ancient astronomy.

We know from equation (5.) that the general function of interpolation for eclipses according to Ptolemy is:

$$m = m_a + (m_p - m_a)\, q(\alpha)$$

Ptolemy applies $m = m_p\, q(\alpha)$ whenever m_a is not greater than zero. However this implies that for some anomalies the resulting magnitude will be greater than zero when in fact it is below zero if we were to extend the line resulting from slope $m_p - m_a$ to such cases in which m_a is actually negative. To be consistent with the rest of cases in which, for a given argument of latitude, there exist both m_p and m_a, it would be necessary to consider negative magnitudes ($m_a < 0$) in the following intervals of the argument of latitude { [77;48°, 79;12°], [100;48°, 102;12°], [257;48°, 259;12°], [280;48°, 282;12°] } in which an eclipse can take place when the Moon is at its perigee, but not when it is at its apogee. In this case, equation (7.) will be as follows:

$$m_a = 21;36^d - 1^d/0;30^\circ \ |\ \omega - 90^\circ\ | \quad \text{with } 77;48^\circ < \omega < 102;12^\circ$$
$$m_a = 21;36^d - 1^d/0;30^\circ \ |\ \omega - 270^\circ\ | \text{ with } 257;48^\circ < \omega < 282;12^\circ \quad (13.)$$

where ω is the argument of latitude. Graphically,

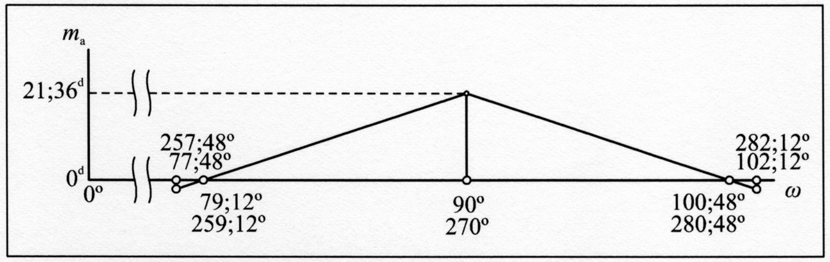

Figure 7. Lunar magnitude as function of the argument of latitude when the Moon is at its apogee.

However, the use of negative numbers as such and of magnitudes not related to physical phenomena was not possible at Jābir b. Aflaḥ's time.

4. Conclusion

In this paper I have studied Jābir b. Aflaḥ's criticism of the computation of lunar eclipses in Ptolemy's Almagest. In this criticism, Jābir b. Aflaḥ claims to have found an error in Ptolemy's computation of lunar eclipses. However, he reaches this conclusion only because there were some missing sections in the manuscript of the *Almagest* he used when he wrote the *Iṣlāḥ al-Majisṭī*. In fact, this is one of the main reasons for Jābir b. Aflaḥ's criticisms of Ptolemy, although not the only one. In addition, he provides a slightly different method for computing lunar eclipses. This method is not a significant improvement from a computational point of view, although, since it avoids interpolation tables, it is far more elegant. However, even though the *Iṣlāḥ al-Majisṭī* is a close rewriting of the *Almagest*, this method for calculating lunar eclipses shows that Jābir b. Aflaḥ deserves to be considered as a creative and original theoretical astronomer.

5. On the edition

What follows is not a critical edition but a working one, based only on the Arabic manuscripts extant in Arabic script. Consequently, we have not used the Arabic manuscripts in Hebrew script, or the Hebrew or Latin manuscripts, although Apianus's Latin edition published in 1534 was consulted during the preparation of this study.

6. Edition

[Es[1] f. 62r, Es[2] f. 74v and B. f. 64r]

في الكسوفات النيرين

وممّا يحتاج إلى تقديمه {في استخراج مقادير الكسوفات وأزمنتها ممّا لم يذكره}[30] هو ما أصفه

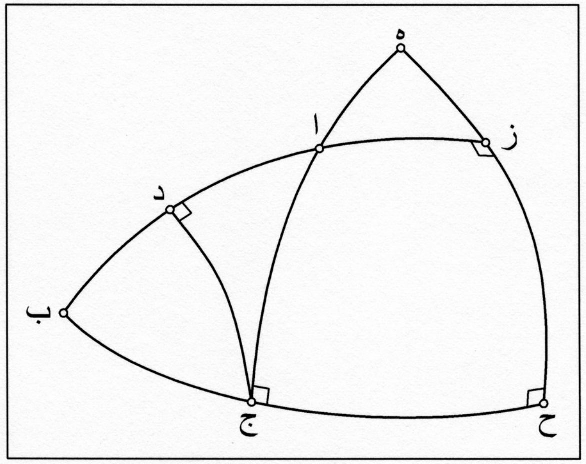

Figure 8. Ms. Es[1] 62v.

[§1] [انظر صورة ٨] فلتكن قوس جيم باء قطعة من فلك البروج وقوس ألف باء قطعة من الفلك المائل للقمر ولتكن نقطة جيم مركز الشمس في الكسوف الشمسي ومركز دائرة الظلّ في الكسوف القمري ولتكن قوس ألف جيم قائمة على قوس باء جيم على زوايا قائمة فتكون نقطة ألف {هي}[31] مركز القمر في

[30] Mss. B., Es[2] ممّا لم يذكره بطلميوس في استخراج مقادير الكسوفات وأزمنتها.

[31] Not in Ms. Es[2].

الاتّصال الحقيقي وقوس ألف جيم عرضه الحقيقي ولتكن قوس {جيم}[32] دال قائمة على قوس ألف باء على زوايا قائمة فتكون نقطة دال هي النقطة التي {يكون عليها}[33] مركز القمر في وسط زمان الكسوف وتكون قوس {دال جيم هي}[34] القوس التي {يكون عليها} مركز النّيرين في وسط زمان الكسوف الشمسي {ومركز}[35] القمر ودائرة الظلّ في وسط زمان الكسوف القمري

[§2] {ومن مقدار}[36] قوس دال جيم يُعلم مقدار المنكسف من قطر المكسوف من النّيرين فأمّا مقدار قوس ألف جيم التي هي عرض القمر في الاتّصال الحقيقي فإنّها تكون معلومة من قبل أنّ قوس ألف باء التي هي بعد القمر من العقدة في الفلك المائل معلومة ونسبة جيبها إلى جيب قوس ألف جيم كنسبة نصف القطر إلى {جيب}[37] نهاية {ميل}[38] الفلك المائل ولما كانت {الثلاثة}[39] جيوب معلومة يكون الرابع وهو جيب قوس ألف جيم {معلوما}[40] {وهي}[41] أصغر من ربع دائرة [Es[2] f. 75r] فهي {إذًا}[42] معلومة

[32] Not in Ms. Es[1].

[33] Ms. Es[2] عليها تكون.

[34] Ms. Es[2] د ج ه.

[35] Ms. Es[2] أو مركز.

[36] Ms. B. وبمقدار.

[37] In the margin in Ms. Es[2].

[38] In the margin in Ms. Es[2]. Not in Ms. B.

[39] Mss. B., Es[2] الثلاث.

[40] Mss. Es[1], B. معلوم.

[41] Ms. B. وهو.

[3§] وأمّا قوس جيم دال فإنّ بطلميوس سامح فيها وجعلها مساوية لقوس ألف جيم {وكذلك}[43] قوس ألف باء جعلها مثل قوس دال باء وذكر أنّ غاية ما تكون {ما}[44] بينهما دقيقتان والذي بينهما نحو أربع دقائق ويمكن معرفة {مقدارها}[45] على التحقيق بأهون سعي

[4§] وذلك أنّا إن أنفذنا قوس ألف جيم حتّى تمرّ بقطب قوس باء جيم وهو {نقطة هاء}[46] ولتكن قوس هاء زاي حاء تمرّ بقطبي {دائرتي}[47] ألف باء [و]باء جيم فتكون كلّ واحدة من زاويتي زاي ودال قائمة وزاوية هاء ألف زاي مثل زاوية جيم [Es^1 f. 62v] ألف دال فتكون ممّا {قدّمناه}[48] نسبة جيب ضلع ألف جيم إلى جيب ضلع جيم دال كنسبة جيب ضلع ألف هاء إلى جيب ضلع زاي هاء وكلّ واحد من جيوب قسي ألف جيم [و]ألف هاء [و]زاي هاء معلوم {فيجب أن}[49] يكون جيب قوس جيم دال معلوما وقوس جيم دال أصغر من ربع دائرة فهي {إذًا}[50] معلومة {وذلك ما أردنا أن نبيّن}[51]

42 Ms. Es^2 إذن.

43 Ms. B. لذلك.

44 Not in Ms. B.

45 Mss. B., Es^2 مقدار قوس ج د.

46 Ms. Es^2 ز.

47 Ms. Es^2 دائرة.

48 Mss. B., Es^2 قدّمنا.

49 Not in Ms. B.

50 Ms. Es^2 إذن.

51 Not in Ms. Es^1. Ms. Es^2 وذلك ما أردنا بيانه.

[5§] [B. f. 64v] وممّا يحتاج إلى تقديمه أيضا هو أن نعلم كيف تستخرج مقادير قطر القمر وقطر دائرة الظلّ في كلّ بعد من أبعاد القمر الذي يكون له من مركز الأرض {من أجل فلك تدويره}[52]

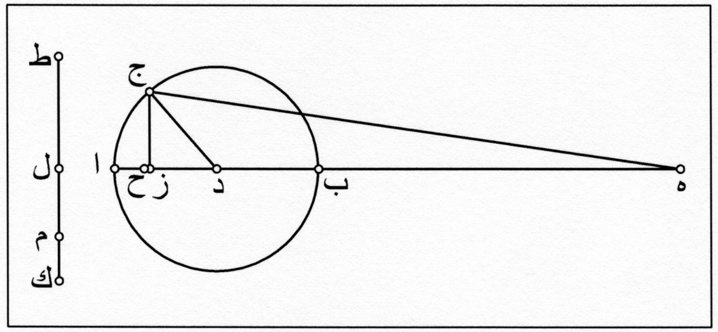

Figure 9. Ms. Es^1 63r.

[6§] [انظر صورة ٩] فليكن فلك التدوير للقمر دائرة ألف باء جيم حول مركز دال وليكن مركز الأرض نقطة هاء ولنصل هاء باء دال ألف فتكون نقطة ألف {هي}[53] أبعد بعد القمر من الأرض في الاتّصالات ونقطة باء أقرب بعده فيها أيضا والفضل بينهما وهو خطّ ألف باء قد تبين أنّه عشرة أجزاء {بالمقدار}[54] الذي به خطّ {هاء دال}[55] ستّون جزءا وقد تقدّم لنا معرفة مقدار قطر القمر ومقدار قطر دائرة الظلّ لكلّ واحد من بعدي ألف هاء [و]باء هاء ولنفرض

52 Mss. B., Es^2 ومعرفة ذلك يكون على ما أصفه.

53 Not in Ms. Es^2.

54 Ms. B. إلى المقدار[!].

55 Ms. Es^2 ه د.

القمر على نقطة جيم من فلك التدوير ونريد أن نعلم مقدار قطره وقطر دائرة الظلّ لبعد هاء جيم فلنخرج عمود جيم زاي فمن أجل أنّ قوس ألف جيم مفروضة وهي قوس الاختلاف في وقت الاتّصال الحقيقي يكون جيبها وهو عمود جيم زاي [Es^2 f. 75v] معلوما {وكذلك}[56] {قوس الاختلاف في وقت الاتّصال الحقيقي}[57] يكون جيبها المنكوس وهو خطّ ألف زاي معلوما يبقى خطّ هاء زاي معلوما فخطّ هاء جيم وهو بعد القمر من مركز الأرض معلوم وليكن {خطّ}[58] هاء حاء مساويا له وليكن قطر القمر لبعد ألف هاء {هو خطّ طاء لام}[59] {وقطره لبعد {خطّ}[60] باء هاء {هو}[61] خطّ طاء كاف والفضل بينهما {هو}[62] خطّ لام كاف}[63] ومقدار قطر القمر لبعد هاء جيم الذي هو مثل خطّ هاء حاء هو خطّ طاء ميم ونريد أن نعلم مقداره فمن أجل أنّ خطّ طاء لام هو مقدار قطر القمر لبعد ألف هاء وخطّ طاء كاف هو[64] مقداره لبعد {باء هاء}[65] والفضل بينهما هو خطّ لام كاف تكون على التقريب نسبة خطّ ألف حاء الذي هو الفضل بين بعدي

56 Ms. B. لذلك.

57 Not in Mss. Es^1, B.

58 Not in Ms. Es^2.

59 Mss. B., Es^2 خط ل ط.

60 Not in Ms. B. Ms. Es^2 بعد.

61 Not in Mss. B., Es^2.

62 Not in Mss. B., Es^2.

63 In the margin in Ms. Es^2.

64 Ar this point, there is a line partially deleted in Ms. B.

65 Ms. Es^2 ه ب.

ألف هاء [و]هاء حاء إلى خطّ ألف باء الذي هو الفضل {بين {بعدي}[66] ألف هاء [و]باء هاء كنسبة خطّ {ميم لام}[67] الذي هو الفضل}[68] ما بين مقداري القطرين {اللذين}[69] لبعدي ألف هاء [و]{هاء حاء}[70] إلى خطّ لام كاف الذي هو الفضل بين مقداري القطرين لبعدي ألف هاء [و]باء هاء وفضل ما بين بعدي {ألف هاء [و]باء هاء معلوم وهو قطر فلك التدوير بأسره {وفضل ما بين القطرين لهما وهو خطّ لام كاف معلوم}[71] وكذلك فضل ما بين بعدي}[72] ألف هاء [Es^1 f. 63r] وحاء هاء وهو خطّ ألف حاء {معلوم}[73] {وخطّ ل ك وهو الفضل بين قطري ط ك [و]ط ل معلوم}[74] فيجب أن يكون خطّ لام ميم وهو الفضل بين القطرين لبعدي ألف هاء وهاء حاء معلوما فنضيفه إلى مقدار القطر لبعد ألف هاء أعني خطّ طاء لام يكون مقدار القطر للبعد المفروض وهو خطّ طاء ميم معلوما {وتمّ بيانه}[75]

66 Not in Ms. Es^1.

67 Ms. B. م ل.

68 Not in Ms. Es^2.

69 Ms. B. الذين. Not in Ms. Es^2.

70 Ms. B. ه ح.

71 Not in Ms. Es^2.

72 Not in Ms. B. In the margin in Ms. Es^2.

73 Ms. B. معلوما.

74 Not in Ms. Es^1.

75 Ms. B. وذلك ما أردنا أن نبيّن وهذه صورته. Ms. Es^2 وذلك ما أردنا بيانه.

[§7] ومثل ذلك بعينه يسلك في مقدار قطر دائرة الظلّ لبعد جيم هاء المفروض

[§8] ولما كان قطر الشمس [B. f. 65r] في جميع {أبعادها}[76] من مركز الأرض لا يتغيّر {كبير التغيّر}[77] لصغر الخطّ الذي يبن مركز الخارج {المركز}[78] لها ومركز فلك البروج وكان قد تبيّن مقداره بأن {وجده}[79] مساويا لمقدار قطر القمر في أبعد بعده في الاتّصالات فكان {لذلك}[80] قطر الشمس معلوما

[§9] [انظر صورة ١٠] فإذا فرضنا قطعة {من}[81] فلك البروج قوس ألف باء وقطعة الفلك المائل للقمر قوس جيم باء ونقطة ألف مركز الشمس في الكسوف الشمسي أو مركز دائرة الظلّ في الكسوف القمري وجعلنا نقطة جيم مركز القمر وقوس ألف جيم مساوية لمجموع نصفي القطرين أعني قطر القمر وقطر الشمس في الكسوف الشمسي [Es^2 f. 76r] أو قطر القمر وقطر دائرة الظلّ في الكسوف القمري ولتكن قوس ألف دال قائمة على قوس باء جيم على زوايا قائمة {وتكون}[82] نقطة دال هي وسط زمان الكسوف وقوس جيم دال من الفلك المائل هي القوس المحيطة بنصف زمان الكسوف وتفصل قوس دال هاء مساوية لقوس

[76] Ms. B. أبعاده.

[77] Not in Mss. Es^1, B.

[78] Not in Ms. B.

[79] Ms. Es^1 وجد.

[80] Ms. B. كذلك.

[81] Not in Ms. Es^1.

[82] Ms. Es^2 فتكون.

جيم دال فتكون قوس {هاء دال}[83] هي أيضا تحيط بنصف زمانه الثاني ومن أجل أنّ عرض القمر في وقت الاتّصال الحقيقي يكون معلوما على ما قدّمنا تكون أيضا قوس ألف دال التي هي المارّة بمركزي الكاسف والمكسوف في وسط زمان الكسوف {معلومة}[84] على ما بيّنّاه آنفا {فنسقطها}[85] من مجموع القطرين يبقى مقدار المنكسف من قطر المكسوف معلوما ومن أجل أنّ خطّ ألف جيم وهو مقدار نصفي القطرين {معلوم}[86] وقوس ألف دال معلوم وزاوية دال قائمة يكون ممّا {بيّنّاه}[87] في مثلثات القسي ضلع جيم دال معلوما

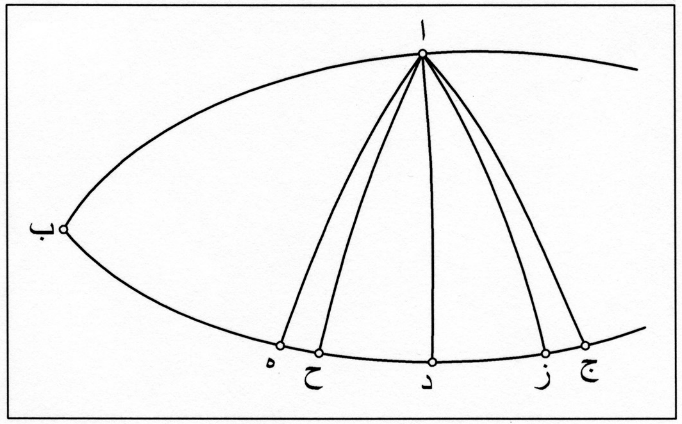

Figure 10. Ms. Es^1 63v.

[83] Ms. Es^2 د ه.

[84] Ms. B. معلوما.

[85] In the margin in Ms. Es^2 خ فنسقطه.

[86] Ms. B. معلوما.

[87] Ms. B. بيّنّا.

[§10] ولما كانت دائرة الظلّ عظيمة يكون للكسوف القمري أربعة أحوال الحالان منها مشتركة للكسوفين {أعني الشمسي والقمري}[88] وهما أوّل الكسوف وآخره والحالان الباقيان ينفرد بهما {القمر وهما}[89] أخر الامتلاء وهو إذا انكسف القمر كلّه {وأوّل}[90] الانجلاء وهو إذا بدأ بالخروج عن دائرة الظلّ

[§11] فليكن مركز القمر في أخر امتلائه نقطة زاي ومركزه في أوّل انجلائه نقطة حاء فتكون قوس جيم زاي على التقريب مثل قوس حاء هاء {وكذلك}[91] قوس زاي دال مساوية على [Es^{1} f. 63v] التقريب لقوس دال حاء ومن أجل أنّ قوس ألف زاي يوتّر نصف قطر دائرة الظلّ تكون معلومة وقوس ألف دال معلومة فقوس زاي دال معلومة وقد كانت قوس جيم دال كلّها معلومة تبقى قوس جيم زاي معلومة فتكون في الكسوف القمري كلّ واحدة من قسي جيم زاي [و]زاي دال معلومة وهي على التقريب مساوية لقسي دال حاء [و]حاء هاء كلّ قوس لنظيرتها

[§12] فبهذا العمل نصل إلى [B. f. 65v] مقدار المنكسف من قطر المكسوف ومقدار {زمان}[92] الكسوف أعني الزمان الذي من أوّل الكسوف الشمسي إلى وسطه {والزمان الذي}[93] من أوّل {كسوف القمر}[94] إلى أخر

[88] Not in Ms. Es^{1}.

[89] Mss. B., Es^{2} الكسوف القمري أحدهما.

[90] Ms. B. والثاني أوّل. Ms. Es^{2} والثاني وهو أوّل.

[91] Ms. B. ولذلك.

[92] Ms. B. أزمان.

[93] Ms. B. والقوس الذي بين وسطه إلى آخره والقوس.

[94] Ms. B. الكسوف القمري.

امتلائه والزمان الذي [Es^{2} f. 76v] من أخر امتلائه إلى وسط زمان الكسوف والذي من وسط زمان الكسوف إلى أوّل انجلائه {والذي}[95] من أوّل انجلائه إلى أخر الانجلاء دون {تقريب}[96] يلحقه

[§13] وأمّا العمل الذي {ذكره}[97] بطلميوس فهو {عمل مقرّب}[98] من وجهين أحدهما أنّه استعمل الخطوط المستقيمة مكان القسي والثاني أنّه جعل القوس التي بين مركزي {الجرمين أعني}[99] الكاسف والمكسوف في وسط زمان الكسوف مثل عرض القمر الحقيقي {في الاتّصال الحقيقي}[100] وذكر أنّ الذي بينهما دقيقتان وهو أكثر من ذلك ثمّ إنّه بعد {هذا}[101] ركّب جدولا لتعديل كسوف القمر على ما أصفه

[§14] وذلك أنّه جمع نصفي قطري القمر ودائرة الظلّ في البعد الأبعد من فلك التدوير وعلم ما يجب لذلك من الدائرة المائلة فوجده عشرة أجزاء وثمانيا وأربعين دقيقة فأسقط ذلك من تسعين {فبقي}[102] تسعة وسبعون {جزءا}[103] واثنتا عشرة دقيقة وهي البعد من النهاية الشمالية فجعلها في السطر الأوّل من سطور

[95] Ms. B. فالذي.

[96] Ms. B. التقريب.

[97] Ms. Es^{2} ذكر.

[98] Ms. Es^{2} على التقريب.

[99] Not in Mss. B., Es^{2}.

[100] Not in Mss. B., Es^{2}.

[101] Ms. Es^{2} ذلك.

[102] Ms. Es^{2} تبقى.

[103] Not in Ms. B.

الجدول الذي للبعد الأعظم وأخذ أيضا مجموع {نصفي}[104] القطرين للبعد الأصغر وعلم ما يجب لذلك من الدائرة المائلة وذلك اثنتا {عشرة}[105] جزءا واثنتا عشرة دقيقة فأسقط ذلك من تسعين بقي سبعة وسبعون جزءا وثمان وأربعون دقيقة فاثبت ذلك في السطر الأوّل من سطور الجدول الثاني الذي للبعد الأصغر وعمل جدولا للدقائق التي نسبتها من ستّين دقيقة كنسبة فضل البعد الأعظم على بعد القمر من الأرض في وقت الكسوف إلى قطر فلك التدوير الذي هو الفضل بين أعظم البعد وأصغره

[15§] {فقال}[106] في تعديل {الكسوفات}[107] أن تدخل بعدد العرض [كذا] في كلّ واحد من الجدولين ويؤخذ ما بإزائه في كلّ واحد منهما وتثبت كلّ واحد على حدته ثمّ تدخل بأجزاء الاختلاف في سطور العدد {الذي}[108] في جدول الدقائق وتؤخذ ما {بإزائها}[109] من الدقائق [Es^{1} f. 64r] فما كانت من ستّين دقيقة أخذ مثل تلك النسبة من فضل ما بين ما أخذ من الجدولين فما كان {ذلك}[110] حمل على ما أخذ من الجدول الأوّل فما كان من الأصابع فهو المنكسف من قطر القمر {وإن}[111] كان عدد العرض لا يوجد [Es^{2} f. 77r] في

[104] Not in Ms. B.

[105] Ms. Es^{2} عشر.

[106] Ms. Es^{2} وقال.

[107] Ms. Es^{2} الكسوف.

[108] Not in Ms. Es^{1}.

[109] Mss. B., Es^{2} بازائه.

[110] Not in Mss. B., Es^{2}.

[111] Ms. B. فان.

الجدول {الأوّل}[112] ولاكن يوجد في الثاني وحده أخذ ما بإزائه منه من الأصابع فكان ذلك مقدار المنكسف من قطر القمر. هذا هو العمل الذي {ذكره}[113]

[16§] فإن كان عدد العرض {تسعة}[114] وسبعين جزءا لم نجدها في الجدول الأوّل ووجدنا بإزائها في الجدول الثاني من الأصابع أكثر [B. f. 66r] من أصبعين

[17§] قلنا إنّ القمر ينكسف منه أكثر من أصبعين وهذا إنّما يكون إذا كان القمر في أقرب قربه من فلك تدويره فإن كان القمر حينئذ في بعده الأبعد أو قريبا منه بحيث يكون مجموع القطرين هناك مثل عرض القمر أو أصغر منه فيكون القمر حينئذ لا ينكسف منه شيء البتّة ونحن قد قلنا إنّه ينكسف منه أكثر من أصبعين وهذه غاية الشناعة أن ينذر بأنّ القمر ينكسف منه أكثر من سدسه ولا ينكسف منه شيء البتّة

[18§] ومثل هذا لا يمكن أن يهمله أحد من واضعي {الزيجات}[115] التي لا برهان عليها فكيف أن يهمله بطلميوس الذي أخذ نفسه بالبراهين الصحيحة على كلّ مطلوب من مطالب هذا العلم حتّى أنّه أخذ نفسه بأن {نبّه}[116] على أنّ الزمان الذي من {أوّل}[117] {زمان}[118] {الكسوف}[119] {الشمسي}[120] إلى وسطه {مخالف

[112] In the margin in Ms. Es^{2}.

[113] Mss. B., Es^{2} ذكر.

[114] Ms. B. سبعة.

[115] Ms. B. الزمان.

[116] Ms. Es^{2} ينبه.

[117] Not in Ms. B.

[118] Not in Ms. Es^{2}.

للزمان}[121] الذي من وسطه إلى آخره وأخذ نفسه بتعديل ذلك ولم يدع التنبيه عليه وإن كان يسيرا جدّا لا يدركه الحسّ فكيف يسوغ لمن {يتحرّى}[122] هذا المقدار من التحرّي {وينبّه}[123] على هذا الأمر الحقير الذي لا يدخل ضررا في شيء من الأشياء ولا يحسّ به أن {يهمل}[124] أمرا يخرج فيه إلى الكذب وأن ينذر بوقوع ما لا يقع البتّة ويفضحه العيان وكان يمكنه الوصول إلى تحقيق ذلك بأهون شيء فهذا ممّا لا يشكّ من أثر الإنصاف {واطّرح المراء والاختلاف}[125] أن هذا الأمر قد ذهب عليه ولم يشعر به لا سيّما {أنّا}[126] {قد}[127] وجدنا له في كتابه {بمعان}[128] قد ذهبت عليه هي أقرب من هذا بكثير فسبحان المنفرد بالكمال {جلّ جلاله وعظم شأنه}[129]

[119] Not in Ms. B.

[120] In the margin in Ms. Es^2.

[121] Mss. B., Es^2 أعظم من الزمان.

[122] Mss. B., Es^2 تحرّي.

[123] Mss. B., Es^2 ونبه.

[124] Ms. B. يحمل.

[125] Ms. B. وأطراح الخلاف والمراد في. Ms. Es^2 وأطرخ المراء والخلاف.

[126] Ms. B. أنّه.

[127] Not in Ms. Es^2.

[128] Ms. Es^1 معان. Ms. Es^2 معانى.

[129] Ms. B. والجلال. Not in Ms. Es^2.

7. Translation

[Jābir b. Aflaḥ's method]

Introductory comments, not mentioned by [Ptolemy], needed to obtain the magnitudes (*maqādir*) and phases (*azmina*) of the eclipses.

[§1] [See Figure 8.] Let arc GB be a segment of the ecliptic and arc AB a segment of the lunar inclined orbit. Let point G be the centre of the Sun at a solar eclipse and the centre of the shadow cone at the lunar eclipse. Let arc AG be perpendicular to arc BG. Point A is the centre of the Moon at the true syzygy while arc AG is its true latitude. Let arc GD be perpendicular to arc AB. Point D is the centre of the Moon at the mid-eclipse. The centre of the Sun and the centre of the Moon at the solar mid-eclipse, or the centre of the Moon and the centre of the shadow cone in the lunar mid-eclipse are on arc DG.

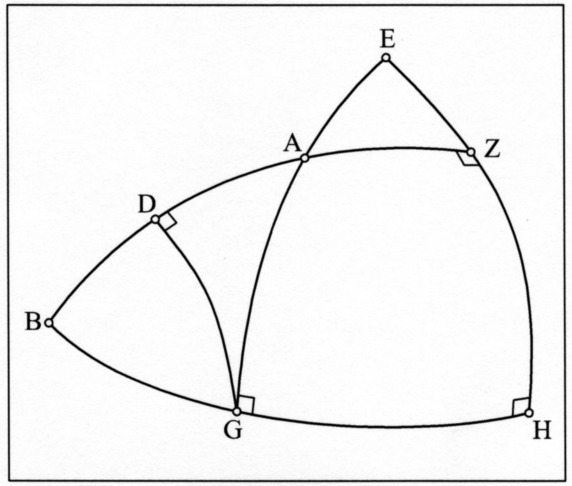

Figure 8. Ms. Es^1 62v.

[§2] From the value of arc DG, the magnitude (*miqdār al-munkasif min quṭr al-maksūf*) of solar and lunar eclipses is known. As for the value of arc AG, which is the lunar latitude at the true syzygy, it is known since arc AB, which is the lunar nodal distance in the inclined orbit, is known. And the ratio of its sine to sine of arc AG is equal to the ratio of the radius to the sine of the maximum inclination of the inclined orbit (*nihāyat mayl al-*

falak al-mā'il). Since the three sines are known, the fourth, which is the sine of arc AG, is known and is less than a quarter of a circle. [Es^2 f. 75r] Therefore, [the arc] is known.

[§3] As for arc GD, Ptolemy acted perfunctorily when he considered it to be equal to arc AG. In like manner, he considered arc AB to be equal to arc DB. He mentioned that the maximum difference between them amounts to two minutes, although the [actual] difference amounts to four minutes. [In addition,] it is easy to know what its actual value is.[130]

[§4] If we extend arc AG until it passes through the pole of arc BG – i.e. point E – and we consider that arc EZH passes through the two poles of circles AB and BG, then angles Z and D are right angles and angle EAZ is equal to angle [Es^1 f. 62v] GAD. Hence, [it can be concluded] from what we have demonstrated previously that the ratio of the sine of side AG to sine of side GD is equal to the ratio of sine of side AE to the sine of side ZE. Since the sines of arcs AG, AE and ZE are known, therefore the sine of arc GD is known. Arc GD is less than a quarter or a circle and, thus, it is known. And that is what we wanted to prove.

[§5] [B. f. 64v] [We must] also consider, in our introductory comments, the way of obtaining the values of the diameters of the Moon and of the shadow cone for all the possible distances of the Moon to the Earth [since] they vary due to the [motion of the Moon in its] epicycle.

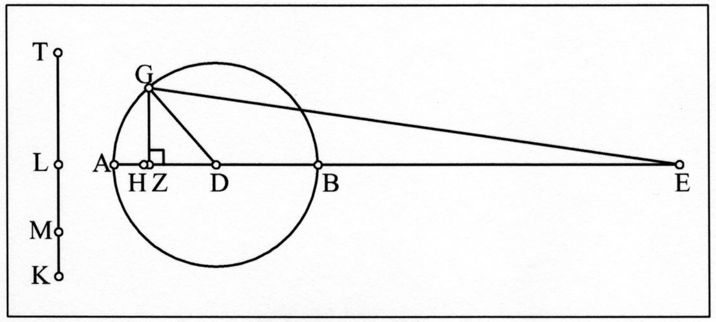

Figure 9. Ms. Es^1 63r.

[§6] [See Figure 9] Let circle ABG around centre D be the epicycle of the Moon and let point E be the centre of the Earth. Let us join [points] E B D A in such a way that point A is the lunar apogee in the syzygies and point

[130] Cf. PtA p. 297-8.

B its perigee in them. Previously we explained that the difference between the two [points], i.e. line AB, is 10^p whenever line ED is 60^p. In addition, we knew the diameter of the shadow cone for distances AE and BE. We will consider as a given condition that the Moon is on point G of its epicycle. We want to know the value of its diameter and the value of the diameter of the shadow cone for distance EG. Let us trace the perpendicular GZ. Since we consider as a given condition arc AG, which is the arc of the anomaly at the true syzygy, [as point G is a given condition], its sine – i.e. the perpendicular GZ – [Es^2 f. 75v] is known. In like manner, the versed sine of the arc of the anomaly at the true syzygy – i.e. line AZ – is known. [Hence], the remaining line EZ is known. Therefore, line EG, which is the distance of the Moon to the centre of the Earth, is known. Let line EH be equal to it, [i.e. to line EG]. Let the diameter of the Moon, for distance AE, be line TL, and its diameter for distance BE be line TK. Therefore, the difference between them is line LK. The lunar diameter for distance EG – which is equal to line EH – is line TM. We want to know its value. Since line TL is the value of the lunar diameter for distance AE, and line TK its value for distance BE and the difference between them is line LK, the ratio of line AH – which is the difference of distances AE and EH – to line AB – which is the difference between AE and BE – is approximately equal to the ratio of line ML – which is the difference between the value of the diameters for distances AE and EH – to line LK – which is the difference between the values of the diameters for distances AE and BE. The difference between distances AE and BE is known and corresponds to the total diameter of the epicycle. And the difference between the diameters for both [distances] – i.e. line LK – is known. In like manner, the difference between distances AE [Es^1 f. 63r] and HE – i.e. line AH – is known; and line LK – which is the difference between diameters TK and TL – is known. Therefore, line LM – which is the difference between the diameters for distances AE and EH – is necessarily known. Thus, [if] we add it [– i.e. line LM –] to the value of the diameter for distance AE – i.e. line TL –, the value of the diameter for the given distance – i.e. line TM – is known. End of the demonstration.

[§7] And exactly in like manner we can apply this method to obtain the diameter of the shadow cone for a given distance GE.

[§8] Since the solar diameter [B. f. 65r] for all [possible] distances to the centre of the Earth does not change due to the small value of the eccentricity – [remember that Ptolemy] obtained its value when he found that it was equal to the value of the lunar diameter when it is on its apogee at the syzygies – [the solar diameter] for all these reasons, was known.

[§9] [See Figure 10] Let us consider as given conditions that arc AB is a segment of the ecliptic, that arc GB is a segment of the lunar inclined orbit, and point A is the centre of the Sun at solar eclipses or the centre of the shadow cone at lunar eclipses. And let us consider that point G is the centre of the Moon and arc AG is equal to the sum of both radius – i.e. the radius of the Moon and the radius of the Sun for solar eclipses, [Es[2] f. 76r] or the radius of the Moon and the radius of the shadow cone for lunar eclipses. Let arc AD be perpendicular to arc BG. Point D corresponds to the mid-eclipse and the arc GD of the inclined orbit corresponds to the half duration of the eclipse. We take arc DE provided that it is equal to arc GD. Therefore, arc ED corresponds to the second half [of the eclipse]. Since the lunar latitude at the true syzygy is known, as we have shown previously, arc AD – which passes through the centres of the eclipsing and eclipsed [bodies] (*kāsif* and *maksūf*) at the mid-eclipse – is known as explained previously. We subtract it from the sum of the two diameters and, thus, the eclipse magnitude (*miqdār al-munkasif min quṭr al-maksūf*) is known. Since [i.] line AG – which is the value of both radii – is known, [ii.] arc AD is known and [iii.] angle D is right, we can conclude from what we explained for spherical triangles that arc GD is known.

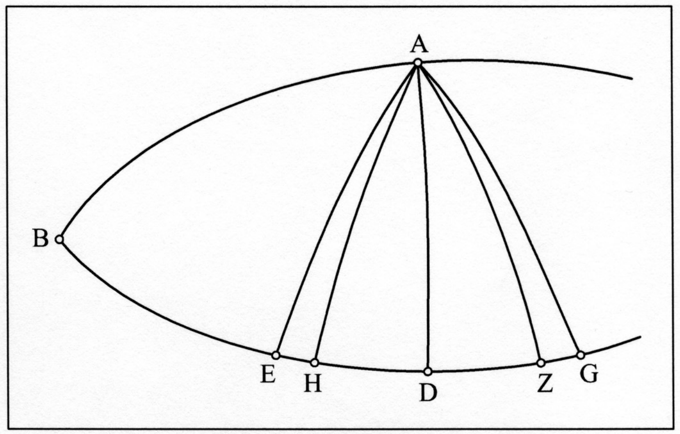

Figure 10. Ms. Es[1] 63v.

[§10] Since the shadow cone is greater [than the lunar diameter], the lunar eclipse goes through four different states (*aḥwāl*). Two of these states are

to be found in both kinds of eclipses – i.e. solar and lunar [eclipses] – which are the beginning and the end of an eclipse. The two other states apply only to the lunar [eclipse]. [These are] the beginning of totality (*ākhir al-imtilā'*) – which is the time when the Moon [begins to be] totally eclipsed – and the end of totality (*awwal al-injilā'*) – which is the beginning of the reappearance of the Moon from the shadow [cone].

[§11] Let point Z be the centre of the Moon at the beginning of totality and point H its centre at the end of totality. Arc GZ is approximately equal to arc HE. In like manner, arc ZD is [Es[1] f. 63v] approximately equal to arc DH. Since arc AZ – which subtends the radius of the shadow cone – is known and arc AD is known, arc ZD is known. And given that the whole arc GD was known, the remaining arc GZ is known. Hence, for the lunar eclipse, arcs GZ and ZD are known and are approximately equal to arcs DH and HE, and therefore each arc corresponds to its counterpart.

[§12] By this procedure we can obtain [B. f. 65v] the eclipse magnitude (*miqdār al-munkasif min quṭr al-maksūf*) and the value of the phases (*zamān*, pl. *azmān*) of the eclipse – i.e. the phase from the beginning of the solar eclipse to the mid-eclipse, the phase from the beginning of the lunar eclipse to the beginning of totality (*ākhir al-imtilā'*), the phase from [Es[2] f. 76v] the beginning of totality to the mid-eclipse (*wasaṭ zamān al-kusūf*), the phase from the mid-eclipse to the end of totality (*awwal al-injilā'*), and the phase from the end of totality to the end of emersion (*ākhir al-injilā'*) – despite the degree of approximation obtained.

[Jābir b. Aflaḥ's criticism of Ptolemy]

[§13] As for the procedure mentioned by Ptolemy, it is an approximate procedure, from two points of view. First, he used straight lines instead of arcs. And second, he considered that the arc between the centres of the two bodies – the eclipsing and eclipsed bodies – at the middle of the eclipse was equal to the true latitude of the Moon at the true syzygy. He considered that [the difference between the two centres under these conditions] was of two minutes. However, [this difference] is greater than this value. After that, he tabulated a table to compute the lunar eclipse which I will summarize here:

[§14] He added the radius of the Moon and the shadow cone when the Moon is considered to be in the apogee of its epicycle and obtained the value for the inclined orbit. He found that this value was 10;48° and subtracted it from 90°, obtaining 79;12°. This value corresponds to the distance to the Northern maximum [nodal] distance [in the inclined orbit in which an eclipse can occur]. He placed this value in the first row of the table which is the maximum distance. In the same manner, he took, the sum of both radii [when the Moon is placed] at its minimum distance [from the Earth] and obtained this value for the inclined orbit, which was 12;12°. He subtracted this value from 90°, obtaining 77;48° and wrote it in the first row of the second table devoted to the minimum distance [of the Moon to the Earth]. He tabulated a table in minutes in which the ratio of these values in minutes relative to 60' is equal to the ratio of the difference between the maximum and minimum distances of the Moon to the Earth at the eclipse relative to the diameter of the epicycle, which is the difference between its maximum and minimum distances.[131]

[§15] On the computation of eclipses he mentioned:

> Enter the value of the [argument in] latitude in both tables and take what you find in front of both [tables] and write it down separately. Enter the degrees of anomaly in the row of the [corresponding] value in the table of minutes and take the value of minutes that you find in front of it. [Es[1] f. 64r] Take a value from the difference of the values obtained from both tables such that its proportion to [this difference] is equal to the [proportion of the minutes obtained relative to 60']. This [proportion] is added to the value obtained from the first table. The result in digits corresponds to the eclipsed section of the lunar diameter. If this value of the [argument of] latitude does not exist [Es[2] f. 77r] in the first table, but only in the second, take the digits that you find in front of it. This [value] corresponds to the magnitude of the lunar eclipse (*miqdār al-munkasif min quṭr al-qamar*).[132]

[131] Cf. PtA p. 296.

[132] If it falls within the range of the numbers in the first two columns, we take the amounts corresponding to the argument of latitude in the columns for the [lunar] travel and the column for the digits [of magnitude] in both tables, and write them down separately. Then, with the anomaly as argument, we enter into the correction table, and take the corresponding number of sixtieths. We then take this fraction of the difference between the [two sets of] digits, [derived from] the two tables, which we wrote down, and also of the difference between the [two sets of] minutes of travel, and add the results of the amounts derived from the first table. If, however, it happens that the argument of latitude falls within the range of the second table only, we take [as final result] the appropriate fraction (determined by the number of sixtieths found [from the correction table]) of the digits and minutes [of travel] corresponding [to the argument of latitude] in the second table alone. The number of digits which we find as a result of the above correction will give us the magnitude of the obscuration, in twelfths of the lunar diameter, at mid-eclipse. Cf. PtA pp. 305-6.

This is the procedure that [Ptolemy] mentioned.

[§16] If the lunar [argument in] latitude were 79°, we will not find [an entry] in the first table. However, in the second table opposite the [argument of latitude of 79°], we will find a value greater than [B. f. 66r] two digits.

[§17] [But] we said that the eclipsed section of the Moon is greater than two digits. And this is only so when the Moon is at the perigee of its epicycle. If the Moon were at the apogee or next to it, in such a way that the addition of the two diameters was equal to the lunar latitude or less than it, no part of the Moon will be eclipsed. But we have stated that it is eclipsed more than two digits. And his pretension that the eclipsed section of the Moon is greater than a sixth is extremely clumsy for it is not eclipsed at all.

[§18] It is impossible that the authors of astronomical tables (*zījāt*) — in which there are no demonstrations — would have neglected this point. And how would Ptolemy have neglected it – a man who intended that all concepts of this science be based upon correct demonstrations, to the point that he drew attention to [such minutiae] as that the time elapsed between the beginning of the solar eclipse and its middle was different from the time elapsed between the middle of the solar eclipse and its end. He intended to compute such matters which are is not worthy of attention since, being so small, they are not perceived by the senses. How is it possible for someone who investigates this value and claims attention to such unnecessary minuteness, which is not perceptible, to neglect an issue that ends in a falsity when he pays attention to an issue that never takes place – and is refuted byh the naked eye. In addition, he could have verified this easily. Whoever claims to be just and does not want to enter into disputes cannot doubt that he neglected this point and did not notice it, particularly when we have found in his book easier questions (*maʿān[in]*) that he had neglected. May the One whose perfection is unique be glorified, His nobility dignified and His issue enlarged.